Climate Shift Uncertainty and Economic Damages

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Draft: November 22, 2026

Climate damage projections typically shift daily temperature distributions by annual-mean warming, overlooking seasonal differences. We quantify the bias by combining daily climate projections with a dose-response function of GDP growth to daily temperatures, estimated from unexpected weather across 1,659 subnational regions. Doing so lowers midcentury annual growth a further 0.052 (0.015–0.088) percentage points, ∼16% of the damage from the naïve mean-shift benchmark. Biases are positive at northern mid-latitudes, where summers outpace annual warming, and negative at high latitudes, where winters do. Globally, they disappear when the benchmark preserves monthly—not annual—mean warming: the omission is seasonal, not day-to-day.

JEL: D62,O44,Q54
Keywords: climate impact, climate risk, climate shift, damage functions, pattern scaling, seasonality

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Knowing how future climate damages might be distributed across time and space is an important research frontier and policy issue for climate scientists and economists alike. Projections of climate damages in macroeconomic models typically rely on reduced-form relationships between climate change and the economy that are based on annual climatic statistics, most often annual mean temperature (Fernández-Villaverde, Gillingham and Scheidegger, 2024). In most integrated climate-economy models, a reduced-form climate module turns emissions into global mean temperature anomalies, further converted into regional annual anomalies via a linear, time-invariant factor, a procedure known as 'pattern scaling' (Leduc, Matthews and de Elía, 2016), before feeding them into damage functions estimated on historical data. These models are global (Nordhaus, 1994; Barrage and Nordhaus, 2024; Cai and Lontzek, 2019; Traeger, 2023), regional (Nordhaus and Yang, 1996), country-level (Nath, Ramey and Klenow, 2024), or gridded (Krusell and Smith Jr, 2022; Cruz and Rossi-Hansberg, 2024; Desmet and Rossi-Hansberg, 2024).[1]

Estimation has gone further than projection. Damage functions have long used intra-annual temperature variation: daily temperatures or degree days for crop yields (Schlenker and Roberts, 2009) and mortality (Deschênes and Greenstone, 2011), and, for aggregate output, seasonal temperatures across US states (Colacito, Hoffmann and Phan, 2019). Sectoral studies also project using the daily,

to Columbia University; computations were performed on the Columbia Business School Research Grid. Replication code available from the authors upon request.

[1] Cruz and Rossi-Hansberg (2024), alone among these papers, uses only winter temperatures (January or July, depending on the hemisphere) rather than annual averages (Lemoine, Hausman and Shrader, 2025).

downscaled output of climate models (Hsiang et al., 2017; Rode et al., 2021; Carleton et al., 2022). Macroeconomic projections do not: by summarizing the projected climate with the change in each region's annual mean temperature, even in subnational studies (Burke and Tanutama, 2019), they assume that warming shifts the whole year by the same amount and leaves the shape of the distribution within the year unchanged. There are three reasons for this simplification. First, carrying the daily output of climate models through many regions, models, and scenarios is computationally costly. Second, the climate modules and emulators of economic models produce annual mean temperature only, so changes within the year cannot be derived for a given emissions path. Third, climate models are trusted less within the year than for the annual mean: their daily output must be bias-corrected (Lange, 2019; Schwarzwald et al., 2026*a*), and in Western Europe, for instance, hot extremes have risen faster than any model simulates (Vautard et al., 2023).

Climate models and observations demonstrate that the shape of the distribution changes. Warming is not uniform across the year: climate models project the amplitude of the seasonal cycle of surface temperature to shrink at high latitudes, where winters warm more than summers, and to grow equatorward (Dwyer, Biasutti and Sobel, 2012). Within seasons, extremes move differently from means: the hottest summer days in North-West Europe have warmed twice as fast as mean summer days (Patterson, 2023), cold extremes warm faster than the mean over North America (Blackport and Fyfe, 2024), and hot days over tropical land

warm more than the average day (Byrne, 2021). Since damage functions are strongly non-linear in daily temperature, a mean shift misses part of the damage. How much it misses, where, through which part of the change in the distribution, and how finely the projected climate must be resolved to capture the effect of the shifting distribution are empirical questions. We answer them here.

We estimate a dose-response function of GDP per capita growth to daily mean temperature across 1,659 subnational regions over 1950–2020, allowing the response to vary with each region's mean climate. We identify the response from temperature surprises, the part of each region's temperature that an autoregressive model of its own past weather does not predict, so that the response is estimated from unanticipated weather rather than from slow-moving differences across regions or over time (Jones et al., 2026). Using five Earth system models (ESMs) of the ISIMIP3b ensemble and three scenarios, we then construct the distribution of daily temperature in three climates: a pre-industrial control, the projected 2050 climate, and a synthetic climate that shifts the control by each region's projected annual mean warming. The synthetic climate is what a projection based on perfect pattern scaling of annual means delivers. The difference in growth effects between the projected and synthetic climates is the bias of the mean-shift benchmark.

We obtain four results. First, the omission is sizable: the projected climate lowers annual growth by a further 0.0519 percentage points (pp) in 2050, 15.7 % of the damage computed from the mean-shifted climate. This represents about

US$34.5 billion a year on 2015 world GDP if the growth effect does not persist, and 1.07 % of world GDP by 2050 if it persists fully. The omitted damage is similar across scenarios, so its share is largest under strong mitigation. Second, the effect derives primarily from the seasonal pattern of warming. To first order, a region's omitted damage are the monthly sum-product of two quantities: how much faster the month warms compared to the annual average, and how much growth the extra warming costs in that particular month. The omission is positive when the months that warm fastest are those in which warming hurts growth, and negative when they are those in which warming helps. Globally, shifting the climate by twelve monthly means instead of one annual mean removes the entire effect, pointing to the importance of seasonality. Day-to-day variability within months works slightly in the opposite direction. Third, the omission forms a dipole: it is positive across the northern mid-latitudes, where summers warm faster than the year when warming hurts, negative at high latitudes, where winters warm faster when warming helps, and close to zero in the tropics. At high latitudes, a mean-shift projection therefore overstates damages; it does not hide gains, since warming still lowers growth in the central estimate. Fourth, North America carries most of the omission under GDP weights; under population weights it spreads to East Asia and the rest of the world. The result is robust to dropping any ESM, to holding the damage function flat beyond its estimation support, and to adaptation to the changing mean climate.

We contribute to three literatures. The first studies how the aggregation of

climate variables affects damage estimates. Waidelich et al. (2024) shows that climate variables beyond annual temperature add to projected damages; Rudik et al. (2022) and Estrada, Tol and Botzen (2025) find larger damages with daily temperatures and seasonality. Winter et al. (2026) also goes beyond the mean, adding the volatility, skewness, and kurtosis of monthly temperature anomalies to a country-level growth regression. But these moments are measured against a fixed 1901–1960 baseline, so they rise mechanically with warming and are hard to separate from the mean. Moreover, they pool all twelve months, making a warm January count like a warm July, which is the seasonal difference that drives our results. Bilal and Känzig (2026) take the opposite route and rely on global annual temperature. We hold the damage function fixed and isolate the projection step: given a daily damage function, how finely must the climate projection be resolved in time? Our answer is that, globally, twelve numbers a year suffice, which is a practical result for the second literature, on climate emulators for economics (Eftekhari et al., 2026): emulating monthly mean warming, which existing emulators can already do (Nath et al., 2022), captures what matters for growth damages, while annual mean warming does not. Third, we contribute to the quantification of uncertainty in damage projections (Rising et al., 2022; Moore et al., 2024; Schwarzwald and Lenssen, 2022): the choice of ESM, rather than the damage function or the scenario, dominates the uncertainty in the omitted damage, because models differ in the seasonal pattern of warming more than in

its level.[2]

Section I presents the climate landscapes, the damage function, and the measure of omitted damage. Section II reports the size of the omission, its mechanism, geographic distribution, and the surprising robustness of our results in light of large overall uncertainties.

I. Climate data, damage function and omitted damage

A. Climate landscapes

We use daily mean near-surface temperature from five ESMs of the ISIMIP3b ensemble (Warszawski et al., 2014): GFDL-ESM4, IPSL-CM6A-LR, MPI-ESM1-2-HR, MRI-ESM2-0 and UKESM1-0-LL, bias-adjusted and downscaled to 0.5° (Lange, 2019), for three shared socio-economic pathways (SSP1-2.6, SSP3-7.0, SSP5-8.5). The ensemble was designed for impact studies and spans much of the CMIP6 range, but five single-member models understate inter-model uncertainty.

For each of the subnational regions of DOSE (Wenz et al., 2023), each ESM and each scenario, we build the distribution of daily temperature (0.1°C bins) in three climates, which we call landscapes. The control landscape pools the piControl simulation, with pre-industrial forcing. The projection landscape pools the ten years 2045–2054 (hereafter '2050') of the scenario run, to approximate the distribution from which the weather of a year around 2050 is drawn.[3] The synthetic

[2]Indeed, Schwarzwald et al. (2026*b*) show that 'observational' uncertainty underpinning ESM projections dominates impact uncertainties. Curtin and Burgess (2026) argue that the climate-economy relationship cannot be credibly identified from historical data. By holding our damage function fixed, our omitted-damage ratio is less subject to this critique.

[3]A longer window would sample more internal variability, for instance more El Niño cycles, at the

landscape shifts every control day by the region's projected change in annual mean temperature in the same ESM: it has the mean of the projection and the shape of the control. We take the control from each ESM's own pre-industrial run rather than from observations so that the control, synthetic and projected climates come from the same model: the omitted damage is a difference between two climates of one ESM, so model biases in the shape of the distribution that are common to them largely cancel, whereas an observed control would count differences between the model and the observations as part of the omission. The pre-industrial forcing also measures warming from the reference point of the temperature anomalies that integrated assessment models use. Grid cells are weighted within each region by population (GPW v4 rev11, 2020; CIESIN, Columbia University, 2018), as in the estimation of the damage function, so that a region's landscape describes the temperatures its population experiences. Appendix A.2 gives the details.

The synthetic climate is a demanding benchmark. It uses each ESM's own regional warming, so it already contains the spatial pattern of warming that pattern scaling aims to reproduce. Whatever it misses comes from changes in the shape of the distribution within the year, not from the spatial distribution of annual warming.

cost of assuming symmetric warming around 2050.

B. *Econometric estimates of climate damages*

The next step combines the climate landscapes with a non-linear dose-response function of GDP growth to daily mean temperature.

For the empirical estimation we combine a global dataset on regional GDP (Database Of Sub-national Economic Output - DOSE v2.14; Wenz et al. (2023)) with climate reanalysis (ERA5; Hersbach et al. (2020)). This gives us a combined climate-economic panel dataset with 1,659 regions from 1950–2020, with temporal coverage of the economic data varying by region. We use real GDP per capita in constant 2015 US$ and calculate annual growth rates. We then apply non-linear transformations to daily temperature and aggregate the daily terms to annual averages, taking into account the fiscal calendar of each country.

When estimating the dose-response function, we account for several recent insights and developments in the climate-econometrics literature. To flexibly account for country-level shocks and trends that could confound the relationship between temperature and GDP growth rates, we include country-year fixed effects in our regression. Furthermore, to account for local and heterogeneous trends across temperature intervals (Jones et al., 2026), we construct temperature shocks from the residuals of location-specific auto-regressive time-series models. In additional robustness tests, we also include several dynamic terms in our regression model (Klosin, 2024). Furthermore, we leverage the cross-sectional richness of our dataset by allowing the effect of temperature shocks to vary by climate. In robustness tests, we also allow for a moderating effect of a region's average income.

We thus estimate a model that relates the growth rate of real GDP per capita in 2015 US$ in administrative unit i in year t as g_{it} to daily temperature levels:

$$(1) \qquad g_{i,t} = \sum_k \Big[\beta_k + \underbrace{\delta_k \, z_i}_{\text{climate (+ income)}}\Big] u_{k,i,t} + \boldsymbol{\gamma}' \mathbf{p}_{i,t} + \mu_i + \nu_{c(i),t} + \varepsilon_{i,t}.$$

The temperature shocks $u_{k,i,t}$ are constructed from the non-linear model terms $b_k\big(T_{i,d}\big)$ of daily temperature $T_{i,d}$ with auto-regressive models:

$$(2) \quad \bar{b}_k(T)_{i,t} = a_{i,k} + \sum_{\ell=1}^{2} \boldsymbol{\pi}'_{i,\ell} \, \bar{\mathbf{b}}(T)_{i,t-\ell} + u_{k,i,t}, \qquad \bar{b}_k(T)_{i,t} = \frac{1}{|Y_{i,t}|} \sum_{d \in Y_{i,t}} b_k\big(T_{i,d}\big).$$

The model includes region fixed effects μ_i and country-year fixed effects $\nu_{c(i),t}$, as well as a non-linear transformation of daily precipitation p_{it}. The error term ϵ_{it} reflects unobserved idiosyncratic shocks to GDP growth and potential measurement error, with standard errors clustered at the country level to account for spatial and temporal dependence. We propagate uncertainty with a wild-cluster bootstrap on the second stage, clustered by country; because each region's first stage uses only that region's data, the country clusters also carry the first-stage estimation error. The projections use the bootstrap replicates as draws of the damage function.

In our main specification we use restricted cubic splines with three knots as non-linear transformation of daily temperature. In robustness checks, we use instead splines with 4 knots and 2nd order polynomials. In additional model

specifications, we include more lags in the auto-regressive model ($l = 3, 4$), lagged terms of the annual growth rate of GDP ($g_{i,t-1}, g_{i,t-2}, g_{i,t-3}$), and lagged terms of the temperature shocks ($u_{k,i,t-1}, u_{k,i,t-2}, ..., u_{k,i,t-10}$). In additional robustness checks, we include location-specific linear time trends or location-specific quadratic time trends instead of using residuals of an autoregressive model as temperature shocks. The results are qualitatively similar.

We find a non-linear dose-response function that agrees qualitatively with earlier work (Burke, Hsiang and Miguel, 2015; Burke and Tanutama, 2019; Nath, Ramey and Klenow, 2024): higher temperature is beneficial for economic growth at cold levels and detrimental at warm levels (Figure 1). It is flatter, though: at 20 °C, a uniform 1 °C warming changes growth by -0.19 pp, against -0.67 in Burke, Hsiang and Miguel (2015) and -1.22 in Burke and Tanutama (2019); a steeper function would scale the omitted and the mean-shift damage alike, leaving their ratio unchanged.

This average response function masks important heterogeneity that is consistent with adaptation to the mean climate: higher temperatures are particularly detrimental for economic growth at warm temperature levels in regions that are generally colder and thus tend to experience fewer of such days (Figure 2).

C. Omitted damage

Let $\gamma_d(T)$ be the effect on region d's annual growth of one year at daily temperature T, evaluated at the region's climatological mean z_d, and f_d^C the distri-

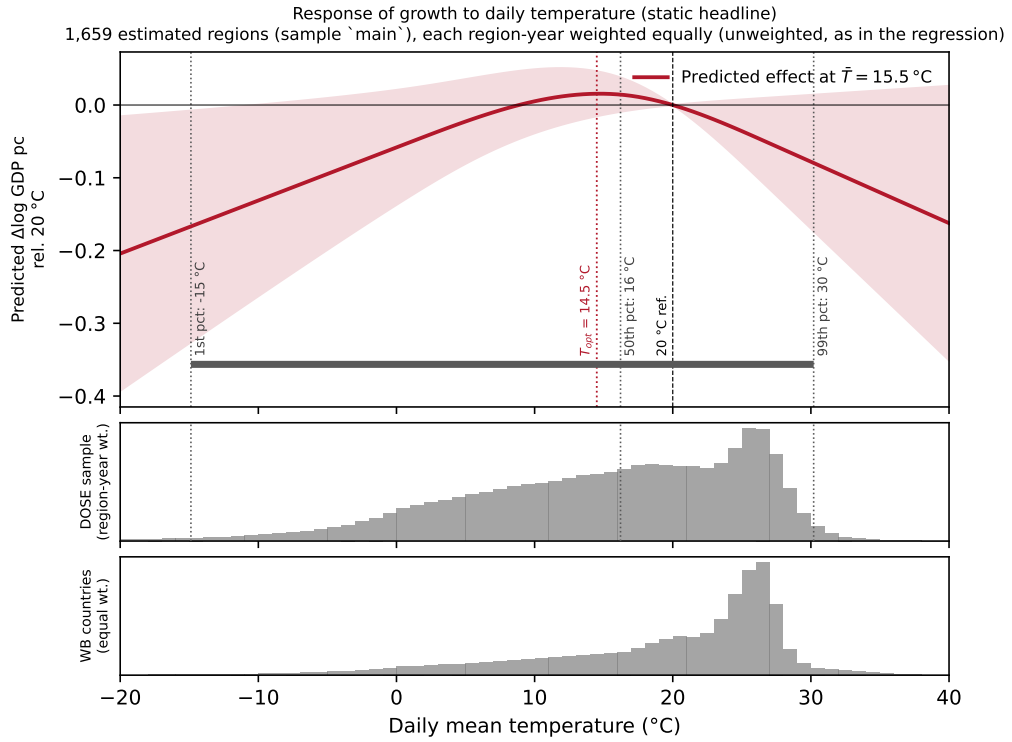

Figure 1. : Response of annual growth rate in GDP pc to daily mean temperature.

bution of daily temperature in landscape C. The growth effect of climate C is $\Omega_d^C = \int \gamma_d(T)\, f_d^C(T)\, dT$, in percentage points of annual growth. The damage a mean-shift projection computes, the naïve damage, is $\Omega_d^{\text{control}} - \Omega_d^{\text{synth}}$; the damage under the projected climate is $\Omega_d^{\text{control}} - \Omega_d^{\text{proj}}$. Their difference is the omitted damage $N_d = \Omega_d^{\text{synth}} - \Omega_d^{\text{proj}}$.

The omitted damage N_d is positive when the mean shift understates the damage. With aggregation weights s_d (2015 GDP shares in the main specification, population shares as the alternative) and a bar for the mean over the 15 ESM×SSP cells, we report the omitted damage $N = \overline{\sum_d s_d N_d}$, the naïve damage

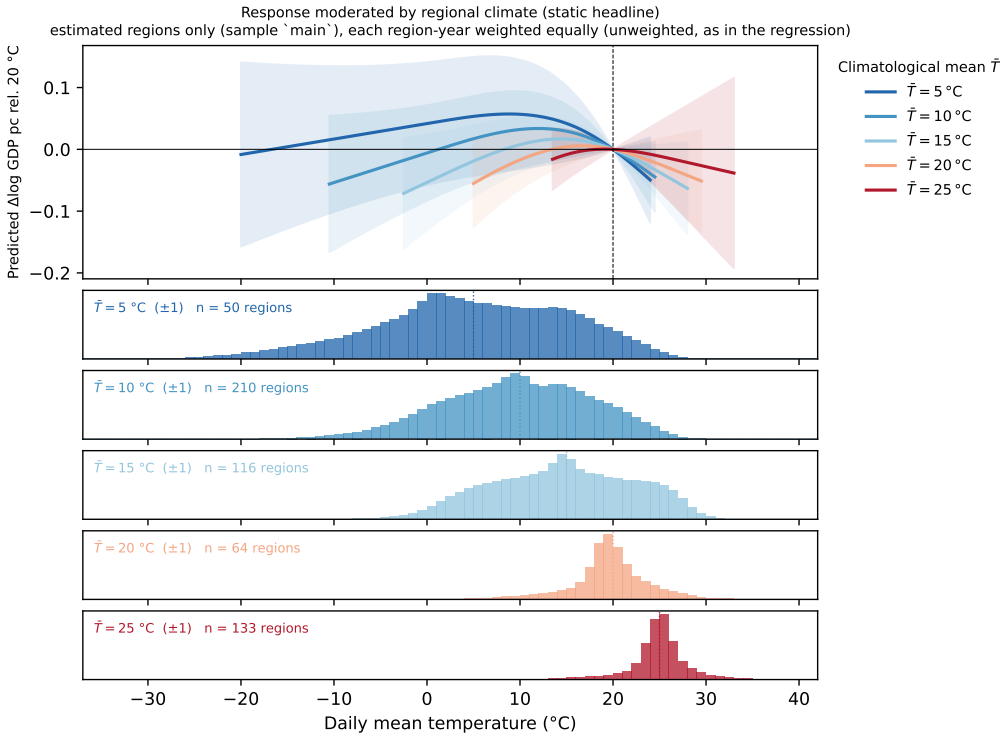

Figure 2. : Response of annual growth rate in GDP pc to daily mean temperature, by mean climate for selected ranges of climatological mean temperature.

$D = \left|\overline{\sum_d s_d(\Omega_d^{\text{synth}} - \Omega_d^{\text{control}})}\right|$, and their ratio $DD = 100\,N/D$. GDP weights make N an effect on world GDP growth: with s_d the regions' shares of GDP at the start of the year, world growth is exactly $\sum_d s_d g_d$, so changes in regional growth rates change it by $\sum_d s_d \Delta g_d$, and holding the shares at their 2015 values is a first-order approximation. Population weights give instead the growth effect experienced by the average person.

We lead with the omitted damage N rather than with the ratio. The naïve damage is the whole effect of warming on growth, which the damage function

does not pin down precisely: across the 999 bootstrap draws of the function it is not always a loss. The omitted damage compares two climates with the same mean and the same number of days, and is estimated much more precisely. Intervals for the ratio therefore vary the numerator only, at the point estimate of the mean-shift damage. All intervals are 5–95 % over the 999 draws. They reflect the uncertainty of the damage function only, with each draw averaged over the climate models; the spread across climate models is reported separately, as the range when one ESM is left out at a time.

II. Results

A. A mean shift understates the damage by about a sixth

Table 1 reports the naïve mean-shift damage, the omitted damage, the damage under the projected climate, and the ratio of omitted to mean-shift damage, each by scenario and by GDP or population weighting. Pooled over the five ESMs and three scenarios, the projected climate lowers annual growth in 2045–2054 by a further 0.0519 pp [0.0145, 0.0881] relative to the mean-shifted climate, 15.7 % [4.4, 26.7] of the naïve damage; equivalently, the mean shift misses 13.6 % [4.2, 21.1] of the damage under the projected climate.

The omitted damage is positive in 99 % of the draws, while the naïve damage is a gain in 27 % of them: the omission is estimated more precisely than the damage it corrects. The omitted damage is of similar size in all three scenarios, while the naïve damage grows with warming, so the share is largest under strong

mitigation. Weighting regions by population more than halves the ratio, because the omission is concentrated in rich mid-latitude regions.

weights	SSP	naïve damage D	omitted N	projected $D+N$	$DD = N/D$ (%)
GDP	SSP1-2.6	0.2430	0.0594 [0.0153, 0.1024]	0.3023	24.4 [6.3, 42.1]
	SSP3-7.0	0.3437	0.0523 [0.0150, 0.0886]	0.3960	15.2 [4.4, 25.8]
	SSP5-8.5	0.4017	0.0440 [0.0126, 0.0746]	0.4457	10.9 [3.1, 18.6]
	all	0.3295	0.0519 [0.0145, 0.0881]	0.3813	15.7 [4.4, 26.7]
population	all	0.4116	0.0282 [0.0081, 0.0479]	0.4399	6.9 [2.0, 11.6]

Table 1—: Naïve, omitted and projected damage, p. p. of annual GDP growth in 2050 relative to the pre-industrial control. Regional mean shift, five ESMs per scenario; 5–95 % over the draws of the damage function (numerator only for DD).

In dollars, one year of omitted growth on 2015 world GDP is US$34.5 billion. If the growth effect were permanent and grew in proportion to global warming from today (Forster et al., 2025) to 2050 (IPCC, 2021, Table SPM.1), GDP in 2050 would be 1.07 % lower. The two figures probably bracket the plausible range between a pure level and a pure growth effect. A version of the damage function with ten annual lags gives the same contemporaneous omission but does not narrow this range: ten years on, the cumulative omitted damage is about two fifths of the contemporaneous one, with an interval that includes both no and full persistence.

B. Mechanism: the seasonal pattern of warming

We establish the mechanism in three steps. First, a second-order expansion shows that the omission is mostly a covariance between the seasonal pattern of warming and the slope of the damage function. Second, building the mean-shift

projection using monthly warming instead of annual warming is enough to avoid a substantial omission. Third, aligning months on each region's warm season shows when in the year it arises.

First, we expand the omission. Write Δ_{dm} for the projected warming of month m in region d, $\bar{\Delta}_d$ for the annual warming, and $\delta_{dm} = \Delta_{dm} - \bar{\Delta}_d$ for how much faster the month warms than the year (so that $\sum_m w_m \delta_{dm} = 0$, with w_m the month's share of days). Let $\bar{g}_{dm}$ and $\bar{h}_{dm}$ be the mean slope and curvature of γ_d over the month's days, and $\Delta\sigma^2_{dm}$ the change in within-month variance. A second-order expansion of the omitted damages around the synthetic climate gives

$$N_d \;\approx\; -\sum_m w_m\, \bar{g}_{dm}\, \delta_{dm} \;-\; \tfrac{1}{2} \sum_m w_m\, \bar{h}_{dm} \left(\delta^2_{dm} + \Delta\sigma^2_{dm}\right). \tag{3}$$

The first term adds up, over the months of the year, the excess warming of each month times the slope of the damage function in that month, with a minus sign: a month that warms faster than the year adds to the omission if warming hurts in that month ($\bar{g}_{dm} < 0$) and reduces it if warming helps ($\bar{g}_{dm} > 0$). Because the excess warming averages to zero over the year, this sum is minus the covariance between the two across months. The second term is the curvature of the damage function times the change in each month's spread around the annual shift; it combines the squared excess warming and the change in within-month variance, which offset each other globally. Under SSP3-7.0 the expansion tracks the exact N_d region by region with $R^2 = 0.998$, and the first-order term carries about nine

tenths of the omission where at least four ESMs agree (Appendix A.3).

Figure 3 shows the mechanism in three regions; the latitude profile on the right shows that they are typical of their bands. Illinois warms fastest in late summer, when the damage function slopes down, and less than the year in winter, when it slopes up: every month adds to the omission, and the twelve terms sum to most of the exact N_d. Southern Finland is the mirror image: its winters warm much faster than the year, when warming helps, and its summers more slowly, when warming would hurt, so a mean shift is too pessimistic there. In Maharashtra, warming is almost uniform across months, the slope of the damage function barely moves over the year, and the monthly contributions are close to zero.

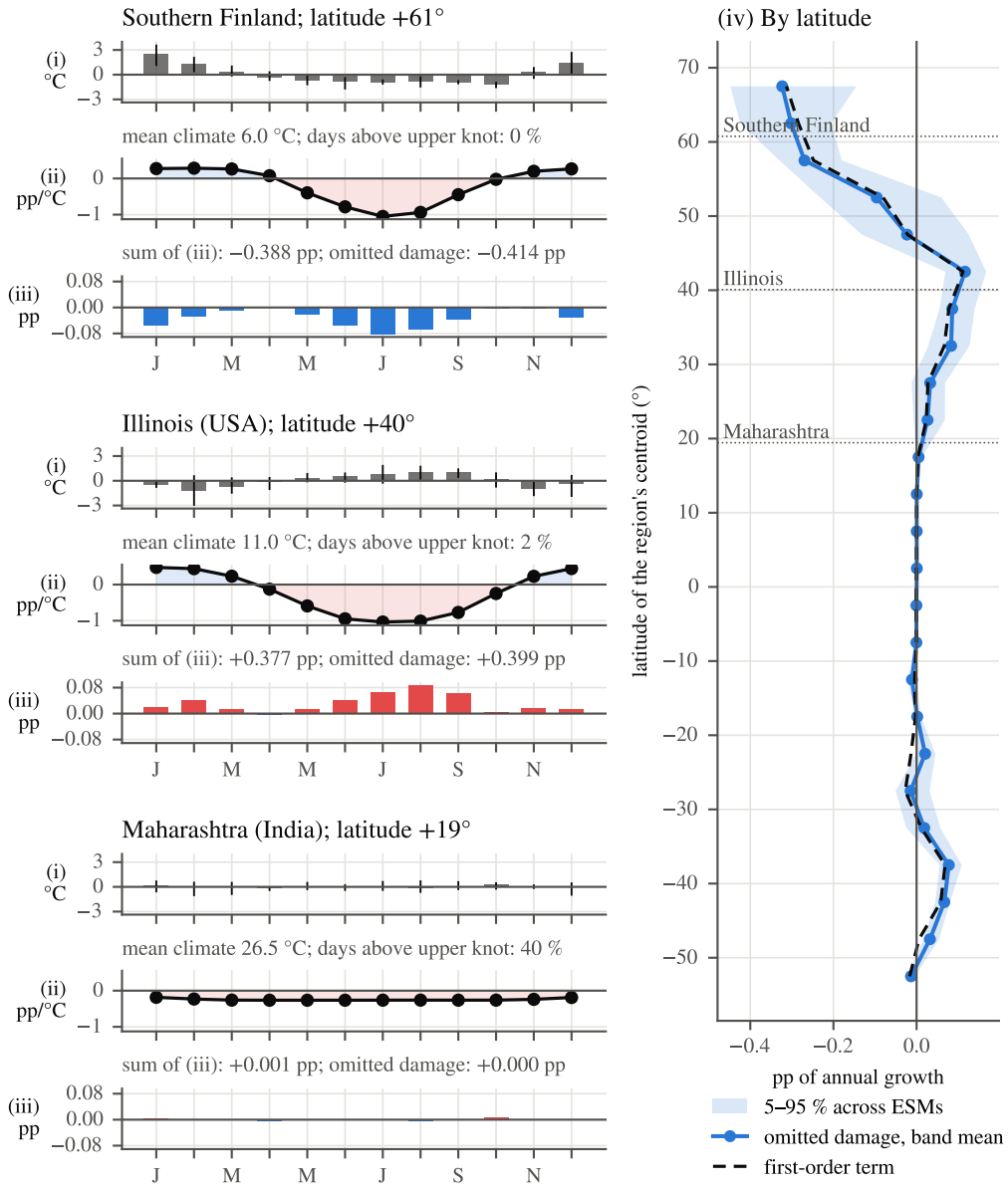

Figure 3. : Where the omission comes from. **Left:** for Southern Finland, Illinois and Maharashtra, (i) how much faster each month warms than the year, δ_{dm}; (ii) the slope of the damage function at the region's own mean climate over that month's days, $\bar{g}_{dm}$; (iii) their product, the month's contribution to the omitted damage. **Right:** omitted damage by 5° latitude band, population-weighted, with the range across ESMs; dashed: the first-order term. SSP3-7.0, 2050.

Second, if the omission comes from the seasonal pattern of warming, a synthetic climate that shifts each month by its own mean warming should remove it. Figure 4 rebuilds the 2050 climate with increasingly fine shifts: one annual, four seasonal, twelve monthly, each applied to the corresponding control days with the damage function evaluated at the shifted temperatures. We call this decomposition stepwise temporal aggregation; it locates the omission in time.

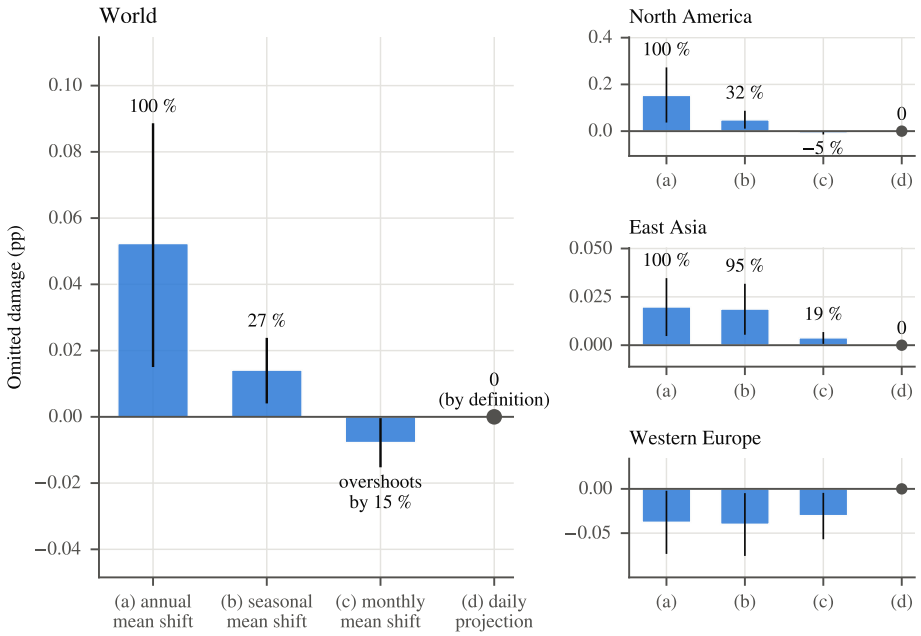

Figure 4. : Omitted damage left when the control climate is shifted by its annual, seasonal or monthly mean warming; the daily projection is zero by construction. SSP3-7.0, GDP weights, 5–95 % over the draws. Labels: share of the annual-shift omission. The Western Europe annual-shift omission is negative.

A seasonal shift removes about three quarters of the omission, and a monthly shift all of it, slightly overshooting. Landscapes built from monthly mean instead of daily temperatures, which keep the seasonal cycle and remove within-month

variability, give a slightly larger omitted damage than daily landscapes. Within-month variability thus works slightly against the seasonal pattern of warming: the damage function is concave at monthly mean temperatures and the projected climate has slightly less within-month variability than the shifted control, mostly in the cold season, consistent in sign with the growth cost of day-to-day variability (Kotz et al., 2021). The monthly shift does not remove the omission everywhere. In Western Europe the omission is negative, and a monthly shift removes only about a fifth of it: there, what the mean shift misses lies mostly in the shape of the distribution within months rather than in the seasonal pattern of warming.

Third, Figure 5 aligns each region's months on its warmest month, thus removing the hemisphere and calendar mix-up of a January–December chart.

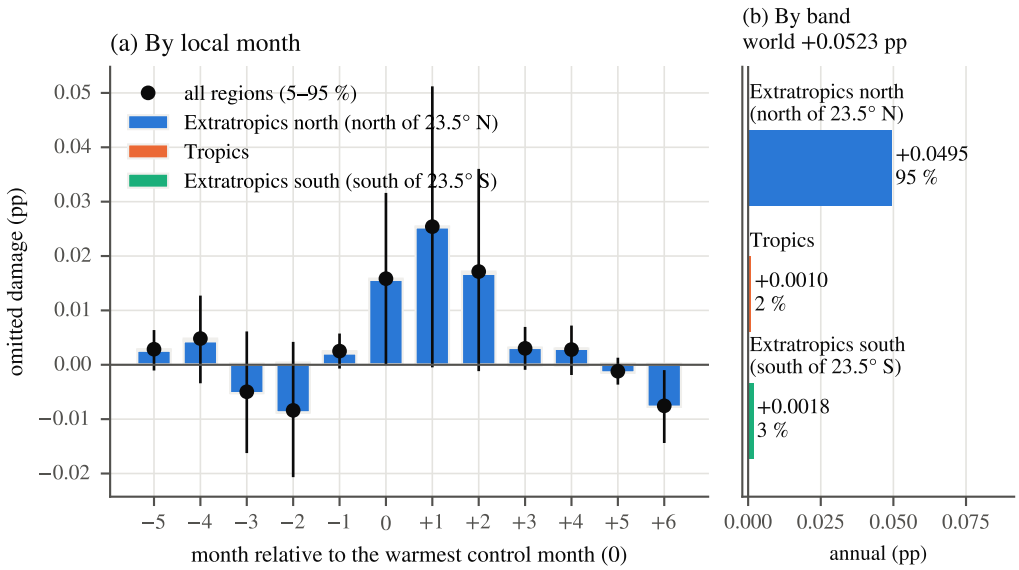

Figure 5. : Omitted damage by month, counted from each region's warmest control month, stacked by latitude band; right: annual total by band. SSP3-7.0, GDP weights, mean of five ESMs, 5–95 % over the damage-function draws.

The omission arises in the warm season and peaks one month after the warmest month: the warmest month and the two that follow account for more than the whole annual omission, and the cold season offsets the excess. Almost all of it comes from the northern extratropics.

C. *Where: a dipole, with the tropics close to zero*

We describe the geography of the omission: first the dipole between the northern mid-latitudes and the high latitudes, then why the tropics are close to zero.

First, Figure 6 maps the regional omitted damage N_d.

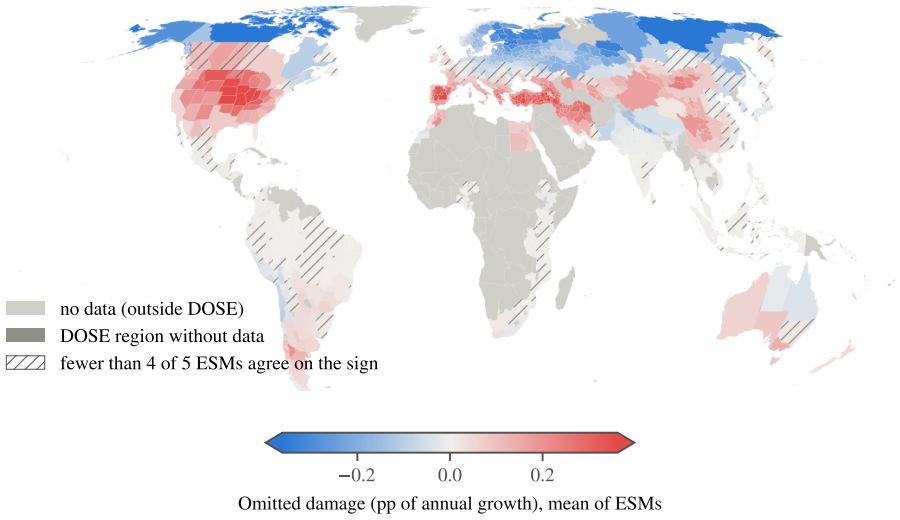

Figure 6. : Regional omitted damage N_d (pp of annual growth), all scenarios, mean of five ESMs. The map does not depend on the aggregation weights. DOSE regions without 2015 GDP are shown but excluded from the GDP aggregation.

Shape changes add damage across the interior and south of the USA, the

Mediterranean from Iberia to Turkey, Central Asia and northern China. They reduce it in Canada, Scandinavia and northern Russia. There, the mean shift overstates damages, but warming still lowers growth in the central estimate, consistent with Fillon (2026). Almost all of the global omission comes from regions where at least four of the five ESMs agree on its sign. Appendix Figure A1 shows what the projection does to the days: at northern mid-latitudes it removes days from the middle of the distribution and adds them on both sides, hotter summers and, relative to the mean shift, cooler winters, both of which cost growth at these climates. Sorting regions by mean climate instead of latitude, all three terciles of z show a positive omission (Appendix Figure A2): the cold tercile mixes Scandinavia with the northern USA and China, which offset each other. The sign follows the seasonal pattern of warming, that is, latitude.

Second, we consider why the tropics are close to zero. By equation (3) the first-order term is the product of three factors: the spread of the slope over the year, the spread of the excess warming, and their correlation. Take Maharashtra in India (Figure 3), the most populous tropical region, which has a strong monsoon cycle. All three factors are small there. The projected warming is almost uniform across months: the excess warming has a spread of 0.12 °C, against 0.7–1.1 °C in Illinois and Southern Finland. Its days sit near or above the upper knot of the damage function, where the function is nearly linear at its climate, so the slope barely moves over the year. And the two are nearly uncorrelated. Only the first factor depends on how the hot tail of the damage function is specified;

holding the function flat beyond its outer knots, the small tropical omission almost disappears. The same holds with four knots.

D. Who misses it

Table 2 states the distribution of omitted damage by continent. North America accounts for three quarters of the omission when weighting by GDP. Europe nets out because it straddles the dipole: the Mediterranean loses, the north gains. Under population weights, East Asia becomes the largest contributor.

	omitted growth, one year (bn US$, GDP weights)	share of global omission (GDP weights, %)	share of global omission (population weights, %)
North America	25.4	74	31
Europe	1.2	3	−4
East Asia	3.2	9	41
Rest of world	4.7	14	32
World	34.5	100	100

Table 2—: Omitted damage by continent, all SSP, mean of ESMs, 2015 GDP.

By income, cutting regions into population terciles of income per capita and computing each group's own ratio of omitted to naïve damage, the poorest third misses nothing, while the middle and richest thirds each miss about a tenth of their damage (Figure 7). Without the USA, the richest third misses much less. The damage function has no income term, so the gradient reflects where each group's regions fall on the dipole, not how vulnerable they are.

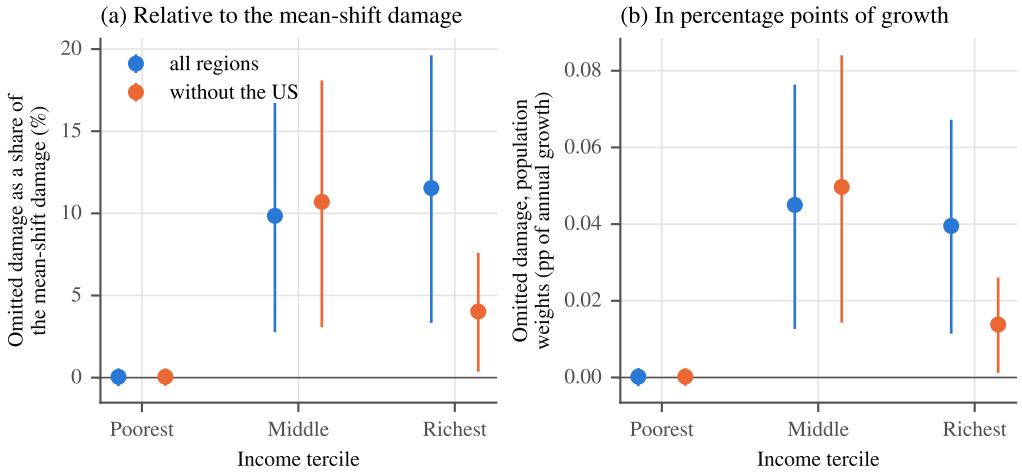

Figure 7. : Omitted damage by income tercile. Regions are cut into terciles of income per capita, each holding a third of the 2015 population (re-cut without the USA). Left: each group's omitted damage as a share of its mean-shift damage (%), the ratio of the two population-weighted means over the 15 ESM×SSP cells. Right: each group's omitted damage, the population-weighted mean over its regions, in percentage points of annual growth. All scenarios, 5–95 % over the damage-function draws (left: numerator only).

E. Robustness and uncertainty

We examine four threats to the validity of our result: the choice of climate model, the extrapolation of the damage function, adaptation to the mean climate, and the coverage of the sample.

First, consider the choice of climate model. Decomposing the variance of the global omitted damage over the damage-function draws, ESMs and scenarios, the choice of ESM explains about half of the variance, more than the damage function or the scenario (Appendix Figure A4). No single model drives the result: dropping one ESM at a time, the ratio stays between 12.1 % and 18.1 %. Nor does the amount of warming order the models: MPI-ESM1-2-HR warms GDP-weighted

regions less but has the second-largest omission, while GFDL-ESM4 has almost none. What differs across models is the seasonal pattern of warming.

Second, consider the extrapolation of the damage function. Holding the damage function flat beyond its outer knots, so that warming outside the estimation support has no effect, keeps 94 % of the global omission. The omission therefore does not come from the hot tail, since the flat function removes every effect of the days above 26.9 °C. Nor can it be assigned to temperature bins: the synthetic and projected climates have the same mean and the same number of days, so adding a linear function of temperature to the damage function, as a change of reference temperature does, moves the contribution of each bin but leaves N_d unchanged. The high-latitude pole leans more on the cold extrapolation than the mid-latitude pole on the hot one. With four knots instead of three, the omitted and the mean-shift damage both rise by about three fifths and the ratio is unchanged.

Third, consider adaptation to the mean climate. The damage function varies with the region's mean climate. Evaluating it at each region's projected 2050 mean climate instead of today's leaves the omission essentially unchanged. The reason is structural: in our estimates the moderation by z mainly tilts the damage function, acting on its linear term, and a linear term integrates to the mean, which the synthetic and projected climates share. Adaptation that changed the curvature of the damage function, for example a hot arm that flattens only beyond some threshold, could change the result.

Fourth, consider the coverage of the sample. DOSE leaves out a fifth of the

world's population, most of it in the tropics, but little of world GDP. Adding the 1,759 units outside DOSE, home to 1.52 billion people, lowers the ratio slightly, to 15.2 % with GDP weights and 6.0 % with population weights (Appendix A.4). Their omission sits in the northern subtropics, from North Africa through the Middle East to Central Asia, and is close to zero in the tropics; it leans more on the extrapolation of the damage function than in DOSE.

III. Conclusion

Damage functions are non-linear in daily temperature and global warming is generally not uniform across the year. Projecting damages by shifting each region's climate by its annual mean warming, the common approach in the literature, understates the growth damage by about a sixth in 2050. The omission is not a matter of extremes or of day-to-day weather variability. It comes from the seasonal pattern of warming, through a simple covariance: damages are understated where the months that warm fastest are those in which warming hurts, and overstated where they are those in which warming helps. This produces a dipole between the northern mid-latitudes and the high latitudes, and leaves the tropics nearly untouched.

Two implications follow. Existing approaches, including pattern scaling even when it is perfect, understate global damages from future climate change. For impact modeling and climate emulation, the fix is cheap: shifting the historical climate by twelve monthly mean warmings instead of the annual mean warm-

ing captures most of what matters for global growth damages. For integrated assessment models, damage functions calibrated on annual mean temperature inherit a bias that is largest, in relative terms, under strong mitigation, and that is concentrated in high-income mid-latitude economies.

Our analysis has limitations. The damage function is estimated on regions with economic data, which under-represent Africa and the tropics; the projection to the rest of the world rests on the moderation of the damage function by the mean climate. Similar gaps exist in the climate data, potentially compounding endemic observational uncertainties (Linsenmeier and Shrader, 2026; Schwarzwald et al., 2026*b*). Given that regions without economic data don't contribute a large share of global GDP, our out-of-sample extrapolation has likely not much effect on our estimated global damages. Furthermore, five single-member ESMs understate the range of projected seasonal patterns of warming, which is the main source of uncertainty in our estimates. We measure effects on growth in a single year; given the short length of our regional GDP panel, our results on the persistence of these effects are ambiguous. The literature suggests a moderate persistence of temperature shocks (Nath, Ramey and Klenow, 2024). Because the regression includes country-year fixed effects, the damage function is identified from a region's growth relative to the rest of its country, so the sum of regional effects need not equal the national effect: shocks that spread to other regions of the same country would make it understate the aggregate damage, and activity displaced toward less affected regions would make it overstate it. Since the omitted

damage applies one function to two climates with the same annual mean, this concerns its level more than its sign or geography, which follow the seasonal pattern of warming, itself resolved only because the regions are subnational. Finally, we consider daily mean temperature only; the same framework applies to daily maximum temperature, humidity or precipitation (Waidelich et al., 2024), and to adaptation that changes the curvature of the damage function rather than its tilt (Mérel, Paroissien and Gammans, 2024; Shrader, 2023; Traeger, 2025).

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Appendix

A.1. Additional results

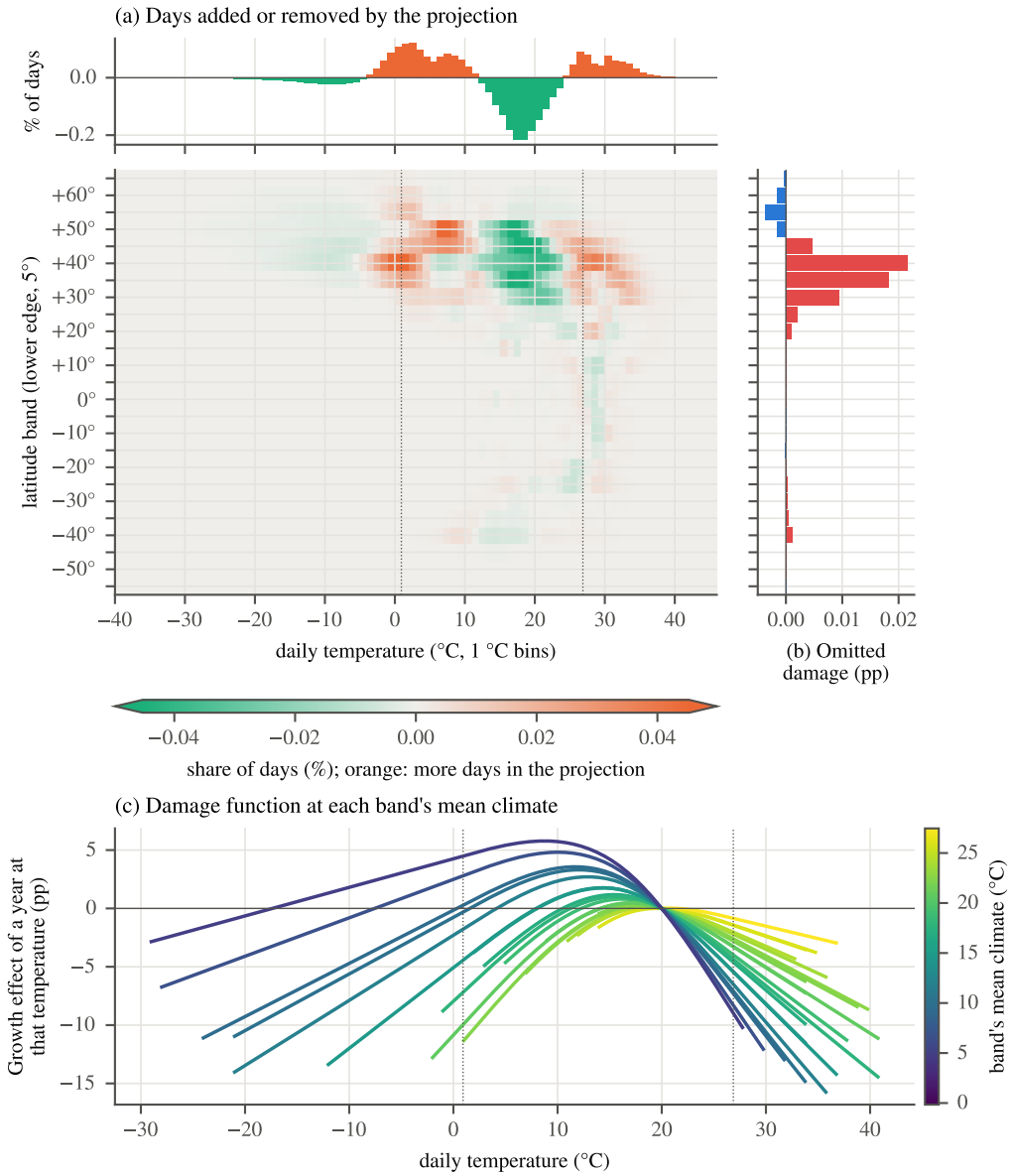

Figure A1. : (a) Days added (orange) or removed (green) by the projection relative to the mean-shifted climate, by daily temperature and 5° latitude band; top: sum over bands. (b) Omitted damage by band, GDP weights. (c) The damage function at each band's mean climate. Mean of the 15 ESM×SSP cells. Panel (a) shows days, not damages: the omission is not identified temperature by temperature.

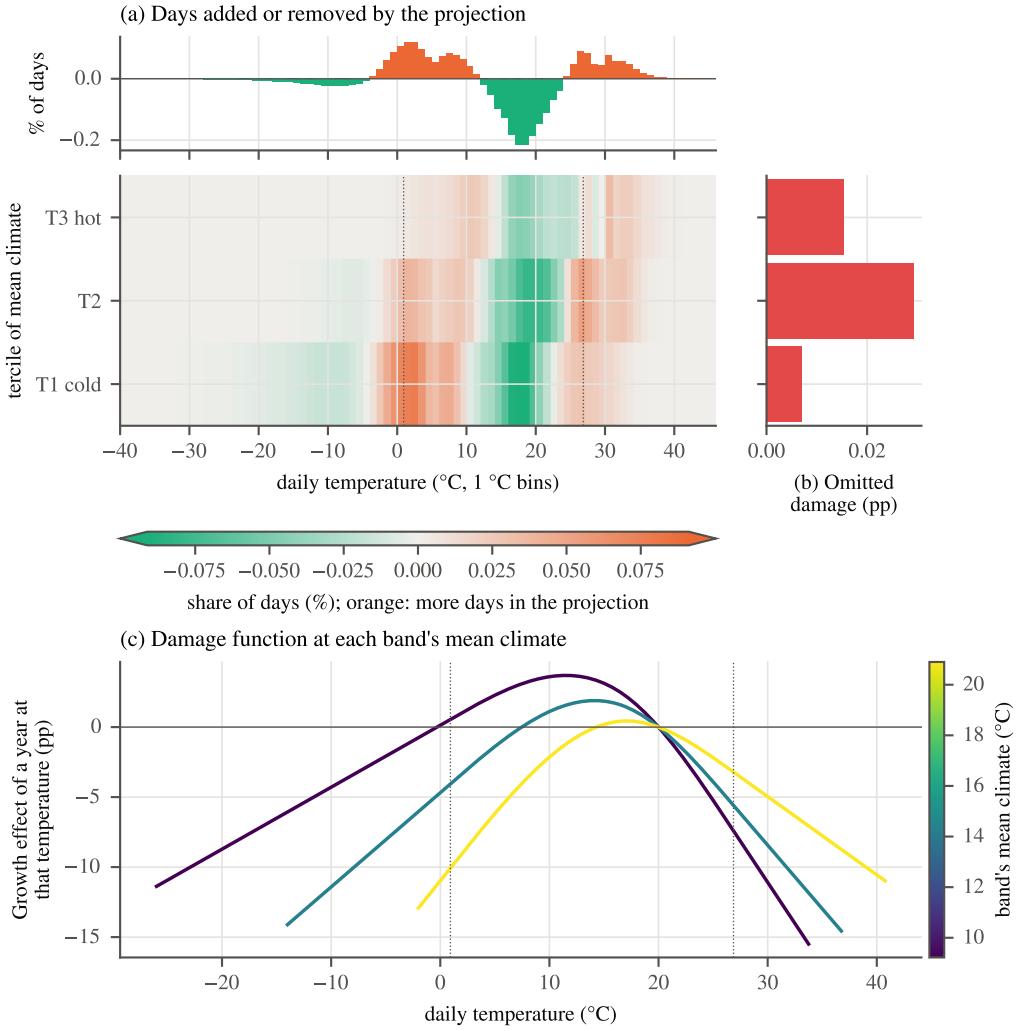

Figure A2. : As Figure A1, with regions grouped by terciles of mean climate z instead of latitude.

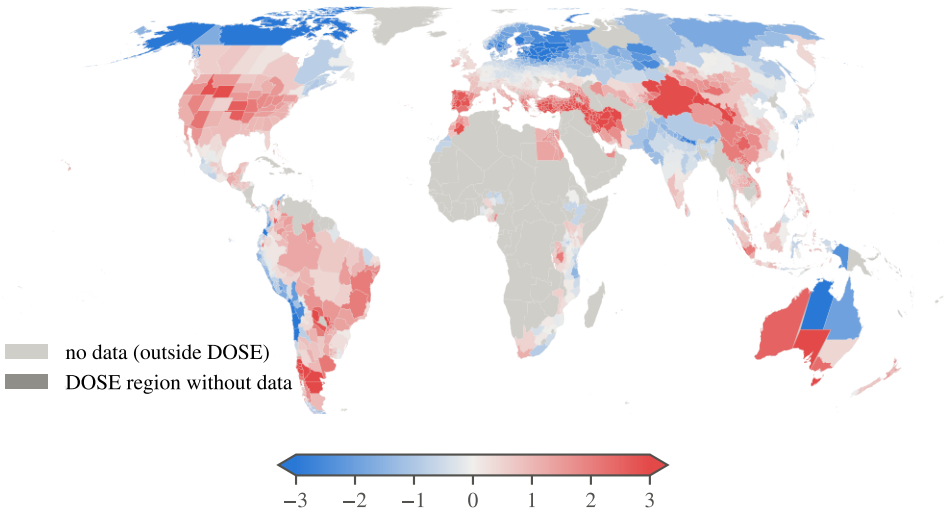

Figure A3. : Signal-to-noise ratio of the regional omitted damage: ensemble mean over the standard deviation across the five ESMs. Regions with $|SNR| > 1$ hold 37.4 % of world GDP and carry 85 % of the global omission. DOSE regions without 2015 GDP are shown but excluded from the GDP aggregation.

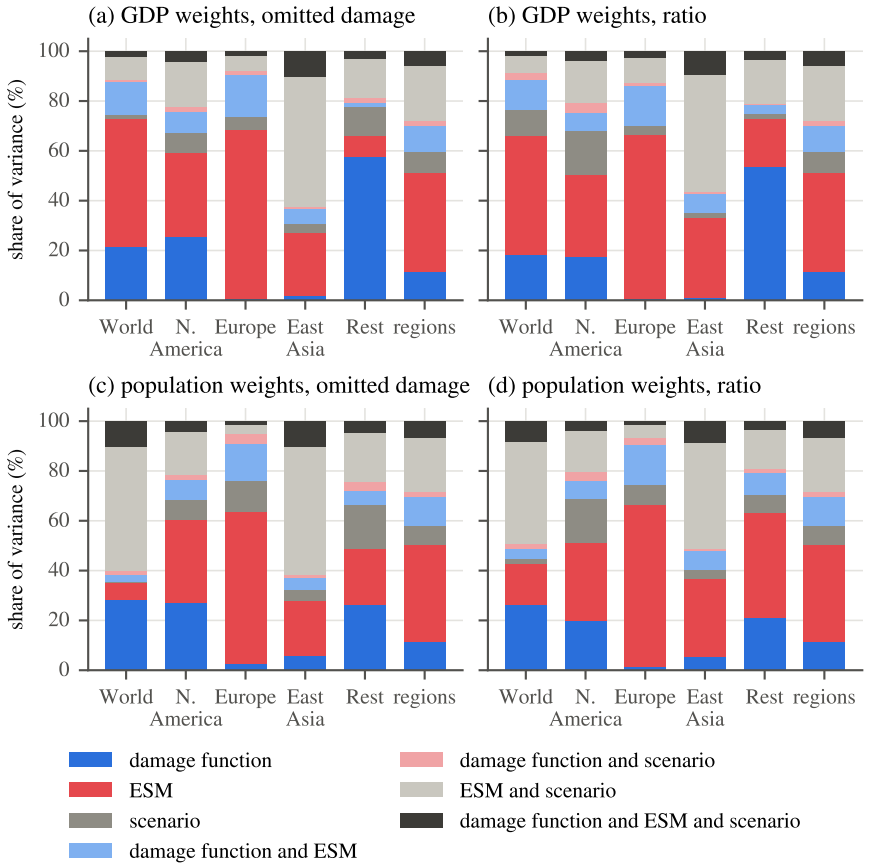

Figure A4. : Variance decomposition of the omitted damage over damage-function draws, ESMs and scenarios, globally and by continent.

A.2. Building the climate landscapes

This appendix describes five elements in turn: the projection window and the control, the cell weights, the synthetic climates, the monthly-grain landscapes, and a fifth step of the resolution ladder. All landscapes are built by one script, run once per ESM and scenario.

First, the projection landscape pools the years 2045–2054 of each scenario run, centred on 2050. The control landscape pools the daily output of each ESM's pre-industrial control run (piControl). Temperatures are converted to degrees Celsius and rounded to the bin width, 0.1 °C.

Second, each DOSE region is linked to the cells of the ISIMIP3b grid that it overlaps. The rows of this key come from the area of each cell inside each region, computed on a finer sub-grid with exact spherical cell areas. The weight of a cell in a region is its GPW v4 rev11 2020 population, assigned with the masks used in the estimation of the damage function: each populated pixel of the population grid goes to the region that contains its centre and to the nearest climate cell, and a region without a populated pixel puts all its weight on the cell nearest its centroid. Each landscape is a histogram of daily temperature in 0.1 °C bins: the frequency of a bin is the sum of the weights of the cell-days that fall in it, pooling days, years and cells, and the histogram is normalised per region before the damage function is applied.

Third, the annual synthetic climate shifts every control day by the region's annual mean warming $\bar{\Delta}_d$ (projection minus control, by ESM and scenario). The

seasonal and monthly versions shift the control days of each season (December–February, March–May, June–August, September–November) or of each calendar month m by that period's own mean warming, Δ_{dm}; the damage function is evaluated exactly at the shifted temperatures, without rebinning. The annual version reproduces the synthetic landscape of the main text.

Fourth, to remove within-month variability while keeping the seasonal cycle, each cell's daily temperatures are replaced by their monthly mean in each year, on the climate grid itself and after leap days are removed, before the weighting and binning. A region's monthly-grain distribution is therefore the population-weighted mixture of its cells' monthly means, not the regional mean temperature of the month: the spread across cells within the region is kept, and only the day-to-day spread within each cell and month is removed. The regional mean warming that defines the synthetic climate is computed from the same monthly values. The daily damage function is applied unchanged and nothing is re-estimated: it is evaluated at each cell's monthly mean, and that value stands for every day of the month. Because the function is non-linear, its value at the monthly mean differs from the mean of its values over the days of the month, and the difference between the daily and the monthly-grain results is this within-month term. These decompositions are accounting identities on the projected distributions: they attribute the omitted damage to time scales of the change in the temperature distribution, and are not an estimated effect of temperature variability on growth. In these landscapes every calendar month has weight 1/12 instead of its share of days;

re-weighting by days moves the omitted damage from 0.0580 to 0.0581 pp.

Fifth, shifting each month by its mean warming and also rescaling its spread to the projected within-month standard deviation leaves a small residual omission, 0.0030 pp $[0.0006, 0.0055]$, in all five ESMs: higher moments of the within-month distribution, and the mixture of cells and years within a region, matter a little.

A.3. Second-order expansion of the omitted damage

This subsection derives equation (3); we drop the region index d. For a landscape C, let W_m^C be the share of the year's days that fall in month m and E_m^C the mean over those days, so that $\Omega^C = \sum_m W_m^C E_m^C[\gamma(T)]$. The synthetic landscape (s) is the control with every day shifted by the annual warming $\bar{\Delta}$, so its months keep the shape of the control's; it gives each month the same share of days as the projection (p), $W_m = w_m$. Insert between the two climates the synthetic month shifted by its own excess warming δ_m:

$$\Omega^p - \Omega^s = \sum_m w_m \Big(E_m^s\big[\gamma(T+\delta_m)\big] - E_m^s\big[\gamma(T)\big] \Big) + \sum_m w_m \Big(E_m^p\big[\gamma(T)\big] - E_m^s\big[\gamma(T+\delta_m)\big] \Big).$$

The first sum moves every day of a month by the same amount. Expanding γ day by day and averaging over the month gives $\bar{g}_m \delta_m + \frac{1}{2}\bar{h}_m \delta_m^2$, where $\bar{g}_m$ and $\bar{h}_m$ are the means of γ' and γ'' over the synthetic month's days, plus terms of third order in δ_m. In the second sum, the shifted synthetic month and the projected month

have the same mean, up to the 0.1°C bins, and differ in their variance, by $\Delta\sigma_m^2$, and in higher moments. Expanding around the common mean, it is $\frac{1}{2}\bar{h}_m^\delta \Delta\sigma_m^2$ to second order, where $\bar{h}_m^\delta$ is the mean curvature over the shifted days. Writing $\bar{h}_m^\delta = \bar{h}_m + (\bar{h}_m^\delta - \bar{h}_m)$ and multiplying by -1, since $N = \Omega^s - \Omega^p$, gives the exact split

$$N = \underbrace{-\sum_m w_m \bar{g}_m \delta_m}_{(a)} \; \underbrace{-\tfrac{1}{2}\sum_m w_m \bar{h}_m \delta_m^2}_{(b)} \; \underbrace{-\tfrac{1}{2}\sum_m w_m \bar{h}_m \Delta\sigma_m^2}_{(c)} \; \underbrace{-\tfrac{1}{2}\sum_m w_m \big(\bar{h}_m^\delta - \bar{h}_m\big)\Delta\sigma_m^2}_{(d)} + (e).$$

Equation (3) is (a) + (b) + (c). The cross term (d) is non-zero only where a month both warms faster or slower than the year and changes its spread; to leading order it is $-\frac{1}{2}\sum_m w_m \bar{k}_m \delta_m \Delta\sigma_m^2$, with $\bar{k}_m$ the mean of γ''', which for a cubic spline is constant between knots. The remainder (e) collects the third-order terms of the shift and the higher moments of the change within months; it is defined as the difference, so the split holds exactly.

Table A1 gives the five terms as shares of N. The first-order term carries most of the omission everywhere outside the tropics. Terms (b) and (c) are of similar size and opposite sign globally, so a form of the expansion that keeps only one of them misses part of the global total even where it tracks the regions closely. The cross term is small, and the remainder is almost entirely the higher moments

	omitted damage N (pp)	(a)	(b)	(c)	(d)	(e)
World, GDP weights	0.0523	93	23	−24	−1	10
World, population weights	0.0280	85	27	−20	−1	9
High latitudes (above 50°)	−0.0796	80	−20	52	4	−16
Mid-latitudes (23.5° to 50°)	0.0853	91	15	−11	0	5
Tropics (up to 23.5°)	0.0087	34	24	37	1	4

Table A1—: Second-order split of the omitted damage. First column: the omitted damage N, in pp of annual growth; (a)–(e): its five terms, as shares of N (%): (a) first-order term; (b) squared excess warming; (c) change in within-month variance; (d) cross term; (e) remainder. Latitude groups by the absolute latitude of the region's centroid, north or south, GDP weights. SSP3-7.0, mean of five ESMs, point estimate of the damage function.

within months.

A.4. Coverage and projection outside DOSE

Outside the DOSE regions live 20.3 % of the world's population. The tropics hold 70 % of the missing population, and 64 % of Africa's population lives outside DOSE; 160 populated countries (1.35 billion people) have no DOSE region. In GDP terms the gap is small: 5.0 trillion US$ in 2015, against 66.4 trillion within the regions with GDP data.

The omitted damage requires no economic data in a region, only its climate and its mean temperature. We therefore extend the projection to the 1,803 GADM level-1 units that contain no DOSE region and have less than half of their population inside DOSE polygons. Their landscapes come from the same script, climate inputs, bins, window, control and population cells as those of the DOSE regions. Their mean temperature z is the 2010–2019 ERA5 mean, computed as for DOSE, and the damage function is the main specification evaluated at each unit's z. We

drop the 35 units without a z, small territories and sea artefacts, and the 13 small islands without a populated cell (4 units are in both groups). That leaves 1,759 units, home to 1.52 billion people. Population weights are GPW 2020 for these units and for DOSE alike. Outside DOSE, GDP is national WDI GDP spread by population.

Table A2 gives the omitted damage and the ratio by continent. Per person, the units outside DOSE omit less than the DOSE regions: 0.0219 pp of annual growth with population weights. Their omission sits in the northern subtropics, from North Africa through the Middle East to Central Asia, and is close to zero in the tropics, sub-Saharan Africa included. There, as in DOSE, warming is nearly uniform across the months of the year. 91 % of the omission outside DOSE lies in the latitude band of Iraq, Syria and Algeria. Adding these units lowers the GDP-weighted ratio from 15.7 % to 15.2 %. With population weights, from GPW 2020 for both sets so that the comparison uses one source, the ratio falls from 6.6 % to 6.0 %. Out of sample, a monthly mean shift removes nearly all of the annual-shift omission (98 % for SSP3-7.0 with population weights), without the overshoot seen in DOSE (Figure 4).

The result outside DOSE leans more on the extrapolation of the damage function. 4 % of the population outside DOSE lives in units whose z is warmer than that of any region in the estimation sample; in DOSE, none does. Already in the control climate, 24 % of their days lie above the upper outer knot of the damage function (26.9 °C), against 19 % in DOSE. In the projection, the share rises to

46 %, against 35 % in DOSE. Holding the damage function flat beyond its outer knots keeps 54 % of the omission outside DOSE, against 83 % in DOSE. With the flat function, the combined GDP-weighted ratio is 17.0 %.

	outside DOSE			DD (DOSE → all)	
	people (M)	N	N, DOSE	population	GDP
Africa	830	0.0124 [0.0034, 0.0212]	0.0256	4.8 → 2.9	5.9 → 6.4
Asia	549	0.0419 [0.0092, 0.0743]	0.0248	6.1 → 6.7	7.8 → 8.7
Europe	11	−0.0190 [−0.0879, 0.0566]	−0.0050	−1.8 → −1.9	4.2 → 4.0
North America	68	0.0007 [0.0001, 0.0013]	0.1004	22.3 → 18.7	29.5 → 28.5
South America	53	0.0027 [−0.0009, 0.0062]	0.0159	2.6 → 2.4	4.6 → 4.5
Oceania	12	0.0008 [0.0002, 0.0015]	0.0489	17.0 → 7.2	15.8 → 14.6
All	1,523	0.0219 [0.0050, 0.0384]	0.0274	6.6 → 6.0	15.7 → 15.2

Table A2—: Omitted damage by continent, DOSE and the units outside it. N: omitted damage (pp of annual growth, population-weighted, with its interval over the damage-function draws); DD: ratio of pooled means (%), each cell from the DOSE regions alone to DOSE and the units outside it (DOSE → all), with population weights (GPW 2020 for both sets) and with GDP weights (DOSE GDP; outside DOSE, national WDI GDP spread by population). Units outside DOSE: those with an omitted damage. All scenarios, mean of five ESMs, main specification.